MOMENTUM AS A USEFUL TSUNAMI DESCRIPTOR

Harold G. Loomis
Honolulu, HI

ABSTRACT

In looking at the videos of the Indonesian tsunami coming ashore at various locations, I thought, "That's a lot of water with a lot of momentum, and that's what does the damage." Perhaps the momentum of a tsunami might be a physical quantity to focus on. Only external forces on the designated body of water create its momentum. Within the body of water, turbulence, internal friction and laminar flow involve internal forces and are not relevant.

This could be particularly useful in the generating area. There could be external forces on a designated body of water from a landslide, a pyroclastic flow, an explosion, from steam generation and from chunks of matter falling into the ocean. The horizontal components of those forces result in horizontal momentums. Ultimately when the wave moves out from the generating area and the internal turbulence and laminar flow get dissipated by friction, in the remaining long wave motion the wave height is simply related to the horizontal momentum. The horizontal momentum contribution to the directionality of the wave would be narrower than that due only to the initial vertical displacement.

Focusing on the momentum description of the tsunami introduces many new kinds of physical problems that are interesting in themselves.

INTRODUCTION

The main idea in this paper is to focus on the momentum of a tsunami. The momentum contained in the largest positive wave crest is a measure of its damaging potential and might be used as a measurement of tsunami magnitude. Since there is little history of thinking about tsunami momentum, many unanswered questions arise. Some are answered quickly with obvious conventional thinking, but others may be difficult. Some momentum questions may lead to worthwhile theoretical considerations. Other momentum questions might be answered with the computer. Giving some thought to questions about momentum would at least broaden the range of considerations that go into tsunami research.

You might look at the tsunami phenomena this way: In one kind of source a vertical uplift distorts the surface of the ocean which results in wave propagation (momentum) in all directions The net horizontal momentum from this is zero, but in any given direction there is a momentum associated with the wave moving in that direction. Add to this the effects of other external forces caused by landslides, horizontal displacements, steam generation and other possible external forces on the designated body of water and additional horizontal momentums are added. Think of a ray from the source to a given location where the effect of the tsunami is to be calculated. Propagate the tsunami along the ray keeping track of changes in the momentum due to external horizontal forces on the water and of the spreading and focusing of momentum due to bathymetric features. This then is the momentum delivered to a destination. It is this momentum which is the agent of destruction.

Momentum is another statement of Newton's Second Law, namely,

$$\vec{F} = m\vec{a},$$

or integrating over time

$$\int \vec{F} dt = \Delta\left(m\vec{v}\right).$$

The impulse imposed on the left, equals the change in momentum on the right. This paper concentrates on the change in momentum on a body of water from the external forces upon it. Let a body of water be specified and $\vec{k}$ be a unit vector in a given direction, then

$$\int \left(\oint \vec{F} \cdot \vec{k}\right) dt = \Delta\left(m\vec{v}\right),$$

where $\vec{v}$ is in the direction of $\vec{k}$ and $\oint$ is the surface integral around the body of water.

GENERATION OF THE TSUNAMI

Landslides. The tsunami of April 1, 1946 has long been an enigma. That tsunami was larger than the size of the earthquake could account for and more directional than most other tsunamis. It was proposed by several researchers that a landslide was part of the generating mechanism. The dimensions of that landslide and the thrust of it have been estimated[1]. The question is (still), what forces are induced on the water by the landslide? I think there are 3 components to these forces which are, in principle, calculable: The first is the leading edge of the landslide pushing against the water in front of it. The second is the turbulent surface of the landslide pushing along the underside of the ocean through friction, and finally, the forces on the water connected with the trailing edge of the landslide. I don't know the solution to any of these problems, but they are three worthwhile topics for research in the tsunami field. Note that the forces due to this landslide are very directional where as the effect of the vertical deformation is less so.

Horizontal displacements. It has been speculated that the whole side of a volcano (viz. Cumbra Vieja[2] in the Canary Islands) collapsing into the ocean would produce a large tsunami. A similar physical problem was when the side of the mountain slid into Lituya Bay in Alaska[3]. I separate this situation out from undersea landslides because one could assume that the total momentum of the earth moving into the water ends up being transferred to the ocean. An insight to this problem can be gained by a simple model. Compare the wave generated by a piston thrusting horizontally into the water with the wave from a piston thrusting vertically into the water. Let the total water displacement be the same in each case and calculate the effect of the additional horizontal momentum in the first case.

Pyroclastic flow. It is not clear how much horizontal momentum is contained in a pyroclastic flow from a volcano, but it is clear that all of that momentum goes into the momentum of the water. Frequently, the pyroclastic flow is not contemporary with the other mechanisms of generation, but when it is, it should be included in the calculations of the resultant tsunami.

Explosions and steam. The heat of a lava flow when it hits the water generates steam. In some cases that steam produces a significant force on the

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[1] Gerard Fryer, personal communication
[2] Pararas-Carayannis (2002b)
[3] Mader

water. A volcano collapsing at or below sea-level is like an explosion[4]. The case of an asteroid hitting the ocean can be treated like an explosion. These clearly impart a horizontal impulse and are a part of the tsunami generation.

MID-OCEAN MOMENTUM CALCULATIONS[5]

In the linear long wave theory, a wave, $\eta(x,t)$, in water depth *h*, has particle velocity *v* given by

$$v = \eta\sqrt{g/h} \ ,$$

so the momentum/unit width is given by

$$M = (h+\eta)\eta\sqrt{g/h} \ .$$

Taking only the first order term, the momentum/unit width in the direction of travel is

$$M = C\eta$$

where

$$C = \sqrt{gh} \ ,$$

is the celerity of the wave! This almost seems intuitively obvious except that the reason for the η is not from the additional wave elevation, but because the particle velocity is proportional to η. One might assume, then, that for any traveling wave form where the particle velocity in the wave is proportional to the wave height above it has the same, or nearly the same rule.

The total momentum/unit width of the tsunami is the integral of *M* along some portion of the wave, say, the first positive crest. For a sinusoidal long wave of semi-amplitude H in water of constant depth *h* and wavelength L is

$$M = C \cdot (2/\pi)HL$$

How does the momentum change during transmission? The bottom friction is a force opposite to the direction of the particle velocity, *v,* which decreases

[4] Mader and Gitling (2006)
[5] Loomis (2002)

the momentum in the part of the wave where η is positive and increases the momentum (i.e. decreases the negative momentum) where η is negative. However, particle velocities are small in deep water and this effect would be small. Only as the particle velocity becomes large as the wave enters shallow water or crosses the shoreline would this be significant.

There is another bottom force, namely the horizontal component of the bottom pressure. In the long-wave linear theory, the wave height is small compared to the depth and the incremental force due to the wave height is negligible is constant along the length of the wave and has no effect. In fact, using just the linear long-wave theory, there is no change in momentum from changes in bathymetry. Why this is so is not entirely clear and needs more thinking. The issue arises again below in the discussion of shoaling.

There is, however, the spreading of the wave front propagating on the surface of a sphere. This accounts for the diminution of wave heights and thus, momentum, with distance.

When the bottom changes across the direction of travel, this produces forces at right angles to the direction of travel which would bend the wave. To the extent that this distorts a straight portion of the wave front into a convex or concave shape it results in scattering or focusing the momentum. This effect is automatically taken care of in calculating the normal propagation of the wave.

How about the change in momentum due to propagation from deep water to the shoreline? In the region where the long-wave linear theory applies, (say approximately $h \geq 10m$), the wave height is inversely proportional to $\sqrt{h}$ and the celerity is directly proportional to $\sqrt{h}$ and these are directly off-setting. So, the momentum is unchanged due to this shoaling.

In other words, the momentum leaving the generating area and headed for a distant shore arrives at an offshore point virtually undiminished except for spreading, small frictional losses, and the cumulative effect of the focusing and spreading effects of bathymetry. When the wave gets into quite shallow water the amplitude and particle velocity become larger and at some point the linear theory is not a good approximation. The definition of "shallow" here depends on the wave height. I would consider a wave with deep water height of 1m. to be treatable by shallow water theory until the water depth gets to be 10m.

Terminal Effects. Consider the total momentum contained in the first crest of a tsunami coming ashore. This momentum is scattered or reversed by forces on the moving water of objects in its path and of the ground itself. These forces are exactly the same but in opposite direction to the forces of the water on the objects, i.e. the destructive force of the tsunami. Thus, the total momentum is a measure of the destructive capability of the tsunami. There are three (at least) kinds of forces to consider. The horizontal forces of the ground on the water (both friction and the horizontal component of pressure) are not destructive. They simply serve to slow the water down and to some extent, reverse the direction of the momentum. The second kinds of forces are those of irregular objects such as brush or trees and junk in general on the water. These forces tend to disorganize the water flow and, in effect, dissipate the organized momentum of the rushing water. The third category of forces are those of buildings, cliff faces, and other large objects. Consider forces on the water which cause the momentum to reverse direction. This would require an impulse that is twice the momentum of the oncoming water. In the process of reversing direction, the wave amplitude is the sum of the incident wave and the reflected wave so that the water level is doubled in amplitude at the reflecting surface. This phenomenon can account for the reports of most large wave heights. In the case of the Indonesian tsunami, the home videos (which certainly pictured well the rushing water and corresponding momentum) showed water levels which judging by its relationship to buildings that it was rushing by of heights of maybe 8 to 14 ft, whereas reported wave heights which would be where the highest water effects were noted were typically double that, or more. The “or more” part of that might be the result of splashes which can reach higher levels.

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