PERSISTENT HIGH WATER LEVELS AROUND ANDAMAN & NICOBAR ISLANDS FOLLOWING THE 26 DECEMBER 2004 TSUNAMI

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ABSTRACT

During the tsunami of 26th December 2004 in the Indian Ocean, media reports suggested that high water levels persisted around the Andaman & Nicobar Islands for several days. These persistent high water levels can be explained by invoking the existence of trapped and partially leaky modes on the shelves surrounding these islands. It has been known in the studies of tides in the global oceans, that there are two distinct types of oscillations, separated in their frequencies by the period of the pendulum day. One species are the gravity waves, and the others are the rotational waves, associated with earth's rotation. Both these species can be found in tidal records around islands as well as near coastlines. Essentially these are either trapped or partly leaky modes, partly trapped on the continental shelves. These two types of modes are usually found in the tsunami records on tide gauges. The tide gauge records as well as visual descriptions of the water levels during and after the occurrence of a tsunami clearly show the presence of these oscillations.

1. INTRODUCTION

The analysis of tidal observations revealed that there are two distinct types of oscillations in the oceans that are separated in their frequencies through the pendulum day. One species are the gravity waves, also referred to as Gravoids modes or Oscillations of the First Class (OFC). The second species are Oscillations of the Second Class (OSC) or rotational modes and are also called Elastoid-inertia oscillations. The first class has gravity explicitly in their frequency equation, while for the second class, which owe their existence to earth's rotation, gravity does not play an important role in their frequencies. The importance of these two types of oscillations in the world oceans is known for a long time, and especially in the tide gauge records near coastlines with wide continental shelves and also in tidal records around islands. The bathymetry of the wide shelf near a coastline or the bathymetry on the shelf surrounding an island can trap a significant amount of long gravity energy associated with such events as tsunamis and storm surges and retain it on the shelf for several days, if not weeks, until the trapped energy slowly leaks away out of the shelf.

The so-called trapped and leaky modes are really the OFC and OSC and can be deduced by analyzing the tide gauge records of tsunamis and storm surges. Specifically in this study we examined the situation of persistent high water levels around the Andaman & Nicobar Islands located in the Bay of Bengal, for the tsunami of 26th December 2004 in the Indian Ocean (Figure 1). Figure 2 shows a detailed map of the Andaman & Nicobar Islands region. Media reports suggested that high water levels persisted around these islands for several days even after the tsunami, and we suggest trapped modes as the most probable cause.

Here we present some theory and some simple analytical calculation which shows that oscillations with periods of few minutes are possible. Observations seem to suggest that most probably this was the case. However, an exact comparison has to wait until we complete a more sophisticated numerical model study is completed as well as a thorough analysis of all the available tide gauge records. But one result that has become obvious even in this preliminary study is that in the trapped modes around Andaman & Nicobar Islands following the tsunami, there were several oscillations with periods of the order of several minutes, and we suggest that these are the Gravoid and Elastoid modes.

Figure 1: Northern part of the Indian Ocean (Courtesy of the University of Texas Libraries, The University of Texas at Austin)

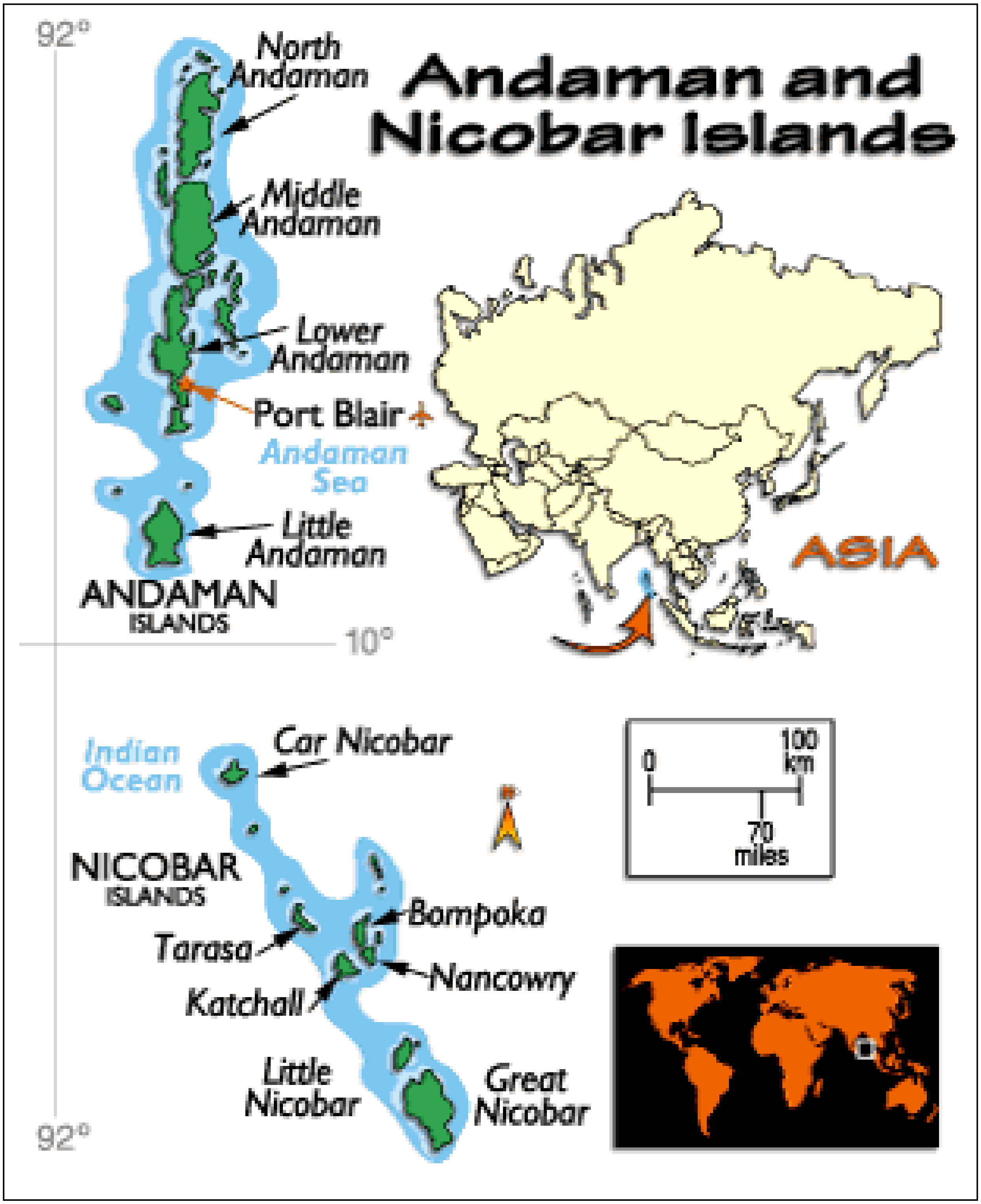

Figure 2: Detailed map of Andaman & Nicobar Islands (www.andamanconnections.com)

2. Oscillations of the first class (OFC) and the second class (OSC)

Laplace (quoted by Lamb, 1932) classified the tidal phenomenon into zonal and tesseral oscillations. In the zonal oscillation, the surface of the liquid is divided into annular zones symmetric with respect to the vertical axis through some hypothetical geometric center. In the tesseral oscillations, the surface of the liquid is divided into a number of compartments, by nodal circles and nodal diameters.

In the tesseral oscillations, Hough (1898) distinguished between OFC (Oscillations of the First Class) and OSC (Oscillations of the Second Class) in the following manner. If σ is the frequency of oscillation, and ω the frequency of rotation, then OFC are those for which:

$$\sigma = \sigma_0 \; (\neq 0) \quad \text{As } \omega \to 0 \qquad (1)$$

And OSC are those for which

$$\sigma \to O(\omega) \quad \text{As } \omega \to 0 \qquad (2)$$

Bjerknes et al. (1934) distinguished between OFC and OSC through the dimensionless ratio $\sigma/2\omega$ as follows:

$$\frac{\sigma}{2\omega}\begin{cases} \geq 1 ....GravityModes(OFC) \\ \leq 1 ....Elastoid - InertiaModes(OSC) \end{cases} \qquad (3)$$

As $\frac{\sigma}{2\omega}$ approaches unity from higher values, gravity modes tend to disappear. For the gravity modes, gravity appears explicitly in the frequency equation. For the rotational modes (Elastoid-inertia oscillations), the frequency for a given mode is a function, mainly of the ratio of the depth of the liquid, to the radius of the container (assumed to be an idealized circular waterbody) and gravity does not play an important role in the frequency equation.

While the intertial stability associated with rotation is a necessary condition for the existence of OSC, it is not a necessary condition for the existence of OFC. Hence, one can ask the question, what is the effect of earth’s rotation on gravity waves, such as tides, tsunamis and storm surges, specifically on their frequencies, besides the well recognized effect of the Coriolis force? Kelvin (1879) partly answered this question in his studies on tidal motions on a rotating earth. He gave the following simple relation:

$$\sigma^2 = \sigma_0^2 + 4\omega^2 \tag{4}$$

Where, ω = Rotation frequency
σ_0 = Frequency of the gravity mode without the influence of earth's rotation
σ = Frequency of the gravity mode when the influence of earth's rotation is included

Eq. (4) is valid when the ellipticity associated with earth's rotation is ignored, in other words, for Eq. (4) to be valid, earth's rotation should have no influence on the free surface of the ocean, i.e. the free surface is always flat, which of course is not the case. However, Eq. (4) is still valid at least in a qualitative sense. It shows that rotation increases the frequency and thus increases the restoring tendency of the system (if the system is perturbed by some external force). However, if the ellipticity is included, this may not always be the case, especially for the higher modes (Murty, 1962).

3. Trapped and Partially Leaky Modes

The trapped and leaky modes concept can be formulated in terms of the OFC and OSC explained in Section 2 (Murty et al., 2005). Eckart (1950) recognized that gravity waves generated in shallow water might get totally reflected while proceeding into deep water at the continental slope. Isacks et al. (1951) used ray theory concepts to show that these waves could be captured from the open sea and guided along a long coastline by successive reflections between the shoreline and the offshore water. Snodgrass et al. (1962) showed from theory that trapped waves could exist along California's borderland, and from spectral analysis of long-wave records they pointed out that the shoreline is a good reflector for long-wave energy. Munk et al. (1964) made observations especially designed to isolate the trapped waves from other motions and concluded that trapped waves account for most low-frequency energy. The important fact that tsunamis approaching a coast can generate trapped waves has been conclusively shown with reference to the Japanese coast by Hatori and Takahasi (1964), Aida (1967), and Hatori (1967). Summerfield (1969) pointed out some unclear use of the terms "trapped waves" and "edge waves".

Stokes (1846), the first to discover these theoretically, used the linearized forms of the surface-wave equations. Ursell (1952) not only found higher-order edge waves but demonstrated their existence in laboratory experiments. Trapping gravity wave energy (caused by guiding waves along the continental shelf) is somewhat analogous to other wave guides in geophysical problems (e.g. Love waves in the earth's crust and sound waves in the Sound Fixing and Ranging (SOFAR) channel in the oceans). An important paper on trapped modes is Longuet-Higgins (1967) who showed that isolated islands might trap long waves that are guided by local bathymetry. The observation that prompted Longuet-Higgins work is the long-wave records at Macquarie Island (54°34′S, 158°58′E) which clearly showed an oscillation period of 6 min and a beat period of 3 h.

The so-called oscillations of the second class (OSC) owe their existence to the earth's rotation. These waves also could get trapped like the gravity waves discussed earlier.

Reid (1958), while studying the effect of Coriolis force on OFC, found another type of very low-frequency motions that propagate along (in the same direction as Kevin waves) and are almost confined to the shoreline, which he called quasi-geostrophic boundary waves. For these waves, the longshore component of the fluid flow and the offshore wave slope approximately balance each other.

Two extreme cases arise: the longshore wavelength of these waves is much less or much greater than the width of the shelf. In the former case, Mysak (1968) showed that the finite width of the shelf has no influence on the waves whereas in the latter case, Robinson (1964) showed that the waves are more or less confined to shallow water and he named these free, second-class, surface wave motions “continental shelf waves.”

The motions considered here under OSC have periods greater than a pendulum day, defined as the period of revolution of a Foucault’s pendulum. This is given by:

$$\text{Pendulum day} = 2T_p = \frac{2\pi}{\Omega \cdot \sin\theta} \qquad (5)$$

Where θ is the latitude and Ω is the angular velocity of earth’s rotation. This expression shows that the pendulum day varies from place to place, depending on latitude. It is $2\pi/\Omega$ at the pole and tends to ∞ at the equator. When the rotation $\Omega \to 0$ the OSC become steady currents. Crease (1956) speculated that in a uniformly rotating ocean of uniform depth the existence of a trapped long wave along a finite length straight-line barrier is possible. The works of Chambers (1964) and Williams (1964) verified Crease’s suggestion. A similar phenomenon was discovered by Sekerzh-Zen’kovich (1968) for a cylindrical island in an ocean of uniform depth. These special types of OSC propagate along the barrier or island in the same sense as a Kelvin wave. However, although the motions exist only in the presence of rotation, they have periods less than a pendulum day and in that sense do not belong strictly to OSC.

4. Computation of the Periods of Trapped Modes

Yanuma and Tsuji (1998) gave the following relationship for edge wave modes trapped on the continental shelf:

$$\omega^2 = \frac{(2N+1)(2M+1)\cdot\pi\cdot\alpha\cdot g}{2L} \qquad (6)$$

Where, α = slope of the shelf
ω = angular frequency of the edge wave mode
g = acceleration due to gravity = 9.8 m/sec^2
L = width of the shelf
N = number of the progressive edge wave mode
M = number of the standing edge wave mode

Period of the edge wave mode = $T = 2\pi/\omega$ (7)

For standing edge wave modes, N = 0
The period of standing edge wave modes is given by:

$$T = \frac{2\pi\sqrt{2L}}{\sqrt{(2M+1)\cdot\pi\cdot\alpha\cdot g}} \tag{8}$$

If we take L = 1 km and α = 1/1000, for the first standing edge wave mode, M = 0

$$T_{S_0} = \frac{2\pi\sqrt{2\times 1000}}{\frac{3.14\times 9.8}{1000}} \text{ seconds} \quad = \frac{2\sqrt{2}\times 3.14\times 1000}{\sqrt{3.14\times 9.8}\times 60} \text{ minutes} = 26.7 \text{ minutes}$$

Table 1 lists the periods for $T_{S_0}, T_{S_1}, T_{S_2}, T_{S_3}$ for different values of shelf width L and shelf slope, α.

Table 1: Periods of the trapped standing edge wave modes for different shelf widths and slopes

Shelf Width L (km)	Shelf Slope α	Modal Number M	Period (Minutes)	Shelf Width L (km)	Shelf Slope α	Modal Number M	Period (Minutes)
1	1/1000	0	26.7	10	1/1000	0	84.38
1	1/1000	1	15.41	10	1/1000	1	48.72
1	1/1000	2	11.93	10	1/1000	2	37.74
1	1/1000	3	10.09	10	1/1000	3	31.89
1	1/100	0	8.44	10	1/100	0	26.68
1	1/100	1	4.87	10	1/100	1	15.41
1	1/100	2	3.77	10	1/100	2	11.93
1	1/100	3	3.19	10	1/100	3	10.09
1	1/10	0	2.67	10	1/10	0	8.44
1	1/10	1	1.54	10	1/10	1	4.87
1	1/10	2	1.19	10	1/10	2	3.77
1	1/10	3	1.01	10	1/10	3	3.19
2	1/1000	0	37.74	20	1/1000	0	119.33
2	1/1000	1	21.79	20	1/1000	1	68.90
2	1/1000	2	16.88	20	1/1000	2	53.37
2	1/1000	3	14.26	20	1/1000	3	45.10
2	1/100	0	11.93	20	1/100	0	37.74
2	1/100	1	6.89	20	1/100	1	21.79
2	1/100	2	5.34	20	1/100	2	16.88
2	1/100	3	4.51	20	1/100	3	14.26
2	1/10	0	3.77	20	1/10	0	11.93

2	1/10	1	2.18	20	1/10	1	6.89
2	1/10	2	1.69	20	1/10	2	5.34
2	1/10	3	1.43	20	1/10	3	4.51
5	1/1000	0	59.67	50	1/1000	0	188.68
5	1/1000	1	34.45	50	1/1000	1	108.94
5	1/1000	2	26.68	50	1/1000	2	84.38
5	1/1000	3	22.55	50	1/1000	3	71.32
5	1/100	0	18.87	50	1/100	0	59.67
5	1/100	1	10.89	50	1/100	1	34.45
5	1/100	2	8.44	50	1/100	2	26.68
5	1/100	3	7.13	50	1/100	3	22.55
5	1/10	0	5.97	50	1/10	0	18.87
5	1/10	1	3.44	50	1/10	1	10.89
5	1/10	2	2.67	50	1/10	2	8.44
5	1/10	3	2.26	50	1/10	3	7.13

For the partially leaky modes, the progressive wave number N will be an integer higher than zero. Table 1 listed the periods for the case N = 0. For the Mode N = 1, the periods can be obtained by dividing the corresponding value in Table 1 by $\sqrt{3}$ or 1.732. For the mode with N = 2, the periods can be obtained by dividing the corresponding value in Table 1 by $\sqrt{5}$ or 2.236. Similar procedure can be used for the higher modes.

CONCLUSIONS

Some theoretical arguments and some simple analytical computations are presented here to explain that the reason for persistent high water levels around the Andaman & Nicobar Islands following the tsunami of 26th December 2004 in the Indian Ocean is the presence of the so-called Gravoid and Elastoid modes of oscillation. These modes are oscillations due to trapping of the long gravity wave energy associated with the tsunami, on the shelf surrounding these islands. While the results presented here are mostly based on theory and some simple analytical calculations, the study is now being expanded to do a proper numerical model as well as analyze all the available tide gauge records. While an exact comparison cannot be made until the numerical model computations are completed, it is worthwhile noting that, the periods of oscillations, of the order of several minutes, in the persistent high water levels around Andaman & Nicobar Islands following the tsunami, that were reported in the media, do agree reasonably well with the results of the simple calculation presented here.

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